Name: Answer Key Hour:_______

Polynomials: Exploring Factors, Roots, and Zeros

This activity examines the connections between the roots or zeros of a polynomial equation and the *x*-intercepts of the graph of the polynomial function. It also looks at how the graph of the function can help identify the factors of the equation

Keeping it Real!!

Part 1: Starting with what we already know!

Graph y = x + 2

1. y = x + 2 is a linear function because the degree is 1.
2. When you graph it, how many x-intercepts are there? 1
3. The x-intercept is x = −2

Graph y = x -3

4. How many x-intercepts are there? 1
5. The x-intercept is x = 3
6. Make a conjecture about the number of x-intercepts of ALL Linear Functions:

All linear functions have 1 x-intercept

What happens if you multiply two linear functions?

7. (x + 2)(x – 3) = $x^2 - x - 6$
8. What is the degree of this new function? 2
9. What kind of function is it? Quadratic
10. Graph the new function. What are the x-intercepts?

x = −2 and x = 3

Complete the table. Use Desmos to help if needed.

Linear Function #1	*Linear Function #2*	Zeros of the linear equations #1	#2	*Quadratic Function*	x-intercepts of the Quadratic Function
(*x* + 4)	(*x* + 3)	−4	−3	$x^2+7x+12$	x = −4, x = −3
(2x + 4)	(x − 2)	−2	2	$2x^2 - 8$	x = −2, x = 2
(x + 5)	(x − 2)	−5	2	$x^2+3x-10$	x = – 5 and x = 2
(3*x* + 3)	(x + 4)	−1	–4	$3x^2+15x+12$	x = −1, x = −4
(2*x* + 4)	(3*x* – 3)	−2	1	$6x^2+6x-12$	x = −2, x = 1
−(x − 2)	(x − 4)	2	4	$-x^2 + 6x - 8$	x = 2, x = 4

Notice that the linear functions are the FACTORS of the quadratic functions.

11. Make a conjecture about the relationships between the factors and the x-intercepts of the quadratic function:

The x-intercepts are the "a" value of (x - a)
Solve the factor by setting equal to zero to get the x-intercepts.

Part 2: Something new!

What happens if you multiply three linear functions?

12. (x + 2)(x – 3)(x + 1) = $x^3 + 2x^2 - 5x - 6$
13. What is the degree of this new function? 3
14. What kind of function is it? Cubic
15. Graph the new function. What are the x-intercepts?
 x = -2, x = 3, x = -1

Complete the table. Use Desmos to graph if needed.

Linear Function #1	***Linear Function #2***	***Linear Function #3***	***Cubic Function***	**x-intercepts of the Polynomial Function**
(x + 4)	(x + 3)	(x – 5)	$x^3 + 2x^2 - 23x - 60$	x = 4, x = -3, x = 5
(x + 5)	(x - 1)	(x - 2)	$x^3 + 2x^2 - 13x + 10$	x = – 5, x = 1, x = 2
(x - 3)	(x + 2)	(x + 4)	$x^3 + 3x^2 - 10x - 24$	x = 3, x = 2, x = -4
(2x + 4)	(3x – 3)	(x + 1)	$6x^3 + 12x^2 - 6x - 12$	x = -2, x = 1, x = -1

What happens if you multiply four linear functions?

16. (x + 2)(x – 3)(x + 1)(x – 4) = $x^4 - 4x^3 - 7x^2 + 22x + 24$
17. What is the degree of this new function? 4
18. What kind of function is it? Quartic
19. Graph the new function. What are the x-intercepts?
 x = 2, x = 3, x = -1, x = 4
20. Make a conjecture about the relationships between the factors and the x-intercepts of polynomial functions in general:

The number of x-intercepts is the same as the number of factors.

21. Make a conjecture about the degree of a polynomial and number of zeros:

The number of zeros is the same as the degree.

Graph the following: $y = x^3 - 9x^2 + 27x - 27$

22. What is the degree of this function? 3

23. How many zeros are there 1

How did that happen!!

If you factor $x^3 - 9x^2 + 27x - 27$, the factored form is $(x - 3)(x - 3)(x - 3)$

$x^3 - 9x^2 + 27x - 27 = (x - 3)(x - 3)(x - 3)$

$x - 3$ is a factor 3 times!

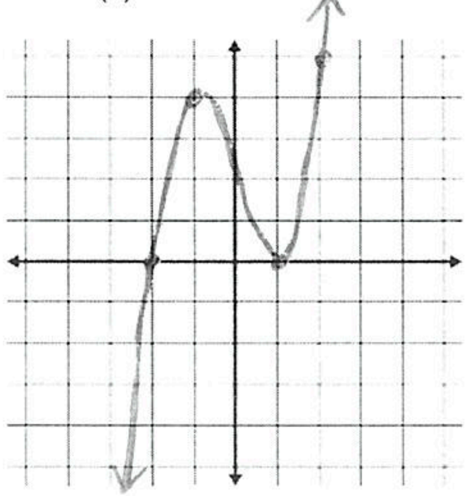
When the same root is a factor multiple times, we call it a **multiple root** and say it has **multiplicity**. The multiplicity of root r is the number of times that $(x - r)$ is a factor of the function.

- ✓ If multiplicity is an even number, the graph touches the x-axis but does not cross. (It makes a U-turn)
- ✓ If multiplicity is an odd number, the graph bends as it crosses the x-axis at that zero.

Graph an example of each:

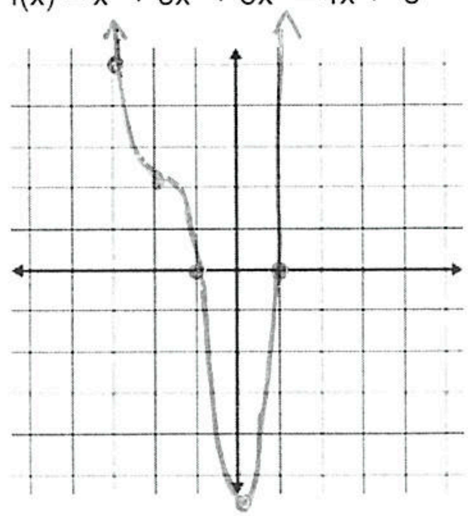
Multiplicity is Even

$f(x) = x^3 - 3x + 2$



Multiplicity is Odd

$f(x) = x^4 + 5x^3 + 6x^2 - 4x$ +- 8



Graph $y = x^3 + x^2 - 8x - 12$

24. What is the degree? 3
25. How many factors will there be? 3
26. What are the factors? X = 3, x = -2
27. What is the multiplicity of each factor? X = 3 has 1 X = -2 has 2
28. Describe the graph at each zero:
graph passes through x-axis at X = -3
Graph makes a U turn at X = -2

Graph $y = -2x^3-12x^2+30x+200$

29. What is the degree? 3
30. How many factors will there be? 3
31. What are the factors? X = 4, X = -5
32. What is the multiplicity of each factor? X = 4 has 1, X = -5 has 2
33. Describe the graph at each zero:
Graph passes through at X = 4
Graph makes U turn at X = -5

Part 3: Graphing is Great- sometimes we need more precision!

Roots can be Rational or Irrational

Finding rational roots is easy- graph it.

To find ALL roots

Step 1: Factor out the rational roots (Synthetic or long division)

Step 2: Use a second method to solve for the irrational roots

- Quadratic formula
- Completing the square
- Using square roots

Rational Root Theorem:.

If the polynomial P(x) has integer coefficients, then every rational root can be written in the form $\frac{p}{q}$ where p is a factor of the constant and q is a factor of the leading coefficient.

To start finding factors, consider all combinations of $\frac{p}{q}$.

Give it a Try:

Problem #1 Find the Rational Roots of $x^3 - 7x^2 + 14x - 6 = 0$

34. What is the degree of the polynomial? 3
35. How many solutions should there be? 3

Step 1: Using Synthetic division, try the factors of $\frac{-6}{1}$. The factors are ± 1, ± 6, ± 2, and ± 3.

(Hint: The factor that works is 3. Factor out x – 3 using synthetic division.)

$$\begin{array}{r|rrrr} 3 & 1 & -7 & 14 & -6 \\ & & 3 & -12 & 6 \\ \hline & 1 & -4 & 2 & 0 \end{array}$$

36. What is the quotient? $x^2 - 4x + 2$

Step 2: Find the solutions to the quotient.

(Hint: Use the Quadratic Formula) $x = \frac{4 \pm \sqrt{16 - 4(1)(2)}}{2}$

$x = \frac{4 \pm \sqrt{8}}{2}$ $x = \frac{4 \pm 2\sqrt{2}}{2}$ $x = 2 \pm \sqrt{2}$

37. What are all the solutions? $x = 3$, $x = 2 + \sqrt{2}$, $x = 2 - \sqrt{2}$

Irrational Root Theorem:.

If the polynomial P(x) has rational coefficients, and a + b$\sqrt{c}$ is a root of the polynomial, the and a - b$\sqrt{c}$ is also a root. Note: a and b are rational and and $\sqrt{c}$ is irrational.

a + b$\sqrt{c}$ and a - b$\sqrt{c}$ always come in conjugate pairs. (You can't have one without the other.)

Problem # 2 Try one on your own.

Find the Rational Roots of $x^3 + 5x^2 - 3x - 3 = 0$

1 Root is 1
Factor is $(x-1)$

38. What is the degree of the polynomial? 3
39. How many roots should there be? 3

$$\begin{array}{r|rrrr} 1 & 1 & 5 & -3 & -3 \\ & & 1 & 6 & 3 \\ \hline & 1 & 6 & 3 & 0 \end{array}$$

$x^2 + 6x + 3$

$x = \frac{-6 \pm \sqrt{36 - 12}}{2}$

$x = \frac{-6 \pm \sqrt{24}}{2}$ $x = \frac{-6 \pm 2\sqrt{6}}{2}$

$x = -3 \pm \sqrt{6}$

Roots: $x = 1$, $x = -3 + \sqrt{6}$, $x = -3 - \sqrt{6}$

Give it a Try:

Problem #1 Find the Rational Roots of $x^3 - 7x^2 + 14x - 6 = 0$

34. What is the degree of the polynomial? ___________
35. How many solutions should there be? ___________

Step 1: Using Synthetic division, try the factors of $\frac{-6}{1}$. The factors are ± 1, ± 6, ± 2, and ± 3.

(Hint: The factor that works is 3. Factor out x – 3 using synthetic division.)

36. What is the quotient? ______________________________________

Step 2: Find the solutions to the quotient.

(Hint: Use the Quadratic Formula)

37. What are all the solutions? ______________________________________

Irrational Root Theorem:.

If the polynomial P(x) has rational coefficients, and $a + b\sqrt{c}$ is a root of the polynomial, the and $a - b\sqrt{c}$ is also a root. Note: a and b are rational and and $\sqrt{c}$ is irrational.

$a + b\sqrt{c}$ and $a - b\sqrt{c}$ always come in conjugate pairs. (You can't have one without the other.)

Problem # 2 Try one on your own.

Find the Rational Roots of $x^3 + 5x^2 - 3x - 3 = 0$

38. What is the degree of the polynomial? ___________
39. How many roots should there be? ___________

Where's Your Imagination?

Part : Adding Complex Numbers to the Mix

Problem #3: Consider ~~$x^3 - 2x^2 - 8x + 52 = 0$~~ $x^3 - 3x^2 + 4x - 12 = 0$

40. What is the degree of the polynomial? 3
41. How many roots should there be? 3
42. Graph the function. How many x-intercepts are there? 1
43. What is the real root? (3,0)
44. What does this tell you about the other roots? they are imaginary!

Use synthetic division to factor the polynomial.

$$\begin{array}{r|rrrr} 3 & 1 & -3 & 4 & -12 \\ & \downarrow & 3 & 0 & 12 \\ \hline & 1 & 0 & 4 & \boxed{0} \end{array}$$

45. What is the quadratic function that is left? $x^2 + 4$

Find solutions to this quadratic function using the Quadratic Formula.

$a = 1 \quad b = 0 \quad c = 4$

$$= \frac{0 \pm \sqrt{(0)^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{0 \pm \sqrt{0 - 16}}{2}$$

$$= \frac{0 \pm \sqrt{-16}}{2}$$

$$= \frac{\pm 4i}{2}$$

$$= \pm 2i$$

46. What are the remaining roots? $2i, -2i$
47. List all the roots of the original polynomial: $3, 2i, -2i$

Problem #4: Consider $x^4 - 3x^3 + 5x^2 - 27x - 36 = 0$

48. What is the degree of the polynomial? 4

49. How many roots should there be? 4

Graph the function and use the Quadratic Formula to find ALL the roots of the polynomial.

$$\begin{array}{r|rrrrr} -1 & 1 & -3 & 5 & -27 & -36 \\ & \downarrow & -1 & 4 & -9 & 36 \\ \hline & 1 & -4 & 9 & -36 & \boxed{0} \end{array}$$

$x^3 - 4x^2 + 9x - 36$

$$\begin{array}{r|rrrr} 4 & 1 & -4 & 9 & -36 \\ & \downarrow & 4 & 0 & 36 \\ \hline & 1 & 0 & 9 & \boxed{0} \end{array}$$

$x^2 + 9 = 0$

$$\frac{0 \pm \sqrt{(0)^2 - 4(1)(9)}}{2(1)}$$

$$\frac{\pm\sqrt{-36}}{2} = \frac{\pm 6i}{2} = \pm 3i$$

50. List all the roots of the original polynomial: $-1, 4, 3i, -3i$

Summary

- ✓ A polynomial of n^{th} degree has how many roots? n
- ✓ Real roots cross the x-axis.
- ✓ Some real roots are irrational. If $a + b\sqrt{c}$ is a root, so is $a - b\sqrt{c}$.
- ✓ Complex number roots come in pairs called complex conjugates
- ✓ If $a + bi$ is a root, then so is $a - bi$.
- ✓ If you know the roots, you can write the factors.
- ✓ If you know the factors you can write the roots.

Mathematical Standards:

- Students will look for and make use of structure (CCSS Mathematical Practice).
- Students will look for and express regularity in repeated reasoning (CCSS Mathematical Practice).

A-APR.2,3,6,7
Understand the relationship between zeros and factors of polynomials

2. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).

3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

F-IF.7b,d
Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.