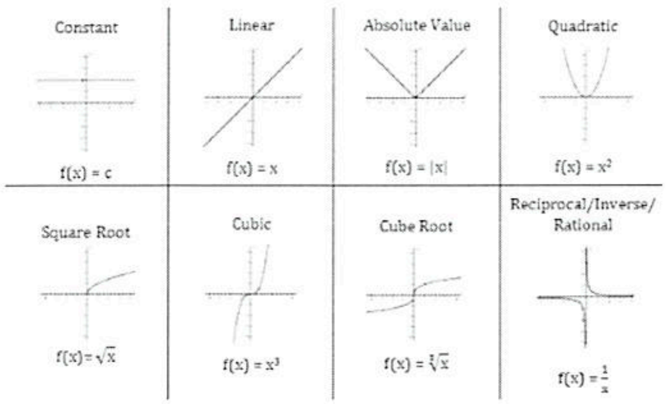
Name: Key

# Transformations of Polynomials

**Parent functions** are the simplest form of that kind of function. Below are some parent functions:




**Transformations** can be described as vertical translations (up and down*), horizontal translations (right and left*), vertical and horizontal stretches or compressions, or reflections.

**Graph** each of the following on your calculator, and compare with its parent function shown above:

| Function | Transformation |
|---|---|
| $y = x^2 + 1$ | Vertical translation of $y = x^2$ up one |
| $y = \lvert x\rvert + 1$ | vertical translation up one (of $y = \lvert x \rvert$) |
| $y = x + 1$ | vertical translation of $y = x$ up one |
| $y = \sqrt{x} - 1$ | vertical translation of $y = \sqrt{x}$ down one |
| $y = x^3 - 0.5$ | vertical translation of $y = x^3$ down 0.5 |
| $y = (x+4)^2$ | horizontal translation of $y = x^2$ left four |
| $y = \lvert x + 3\rvert$ | horizontal translation of $y = \lvert x \rvert$ left three |
| $y = (x-1)^3$ | horizontal translation of $y = x^3$ right one |
| $y = \sqrt{x-2}$ | horizontal translation of $y = \sqrt{x}$ right two |
| $y = -\lvert x\rvert$ | reflection of $y = \lvert x \rvert$ over x-axis |
| $y = -\sqrt{x}$ | reflection of $y = \sqrt{x}$ over x-axis |
| $y = (-x)^3$ | reflection of $y = x^3$ over y-axis |
| $y = \sqrt{-x}$ | reflection of $y = \sqrt{x}$ over y-axis |

*Because you are all sophisticated individuals, you no longer should use the terms “up”, “down”, “left”, or “right”, but rather “horizontal translation to the left/right” and “vertical translation up/down”

**Generalize** the behavior you observed for any function :

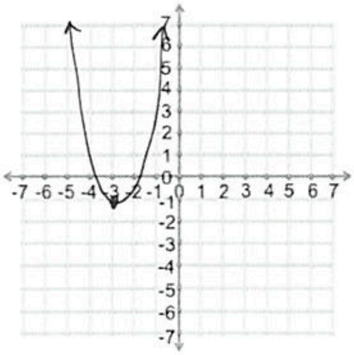
| *Function Notation* | *Transformation* |
|---|---|
| $f(x)+c$ | vertical translation up c |
| $f(x)-c$ | Vertical translation down c |
| $f(x-c)$ | horizontal translation right c |
| $f(x+c)$ | Horizontal translation left c |
| $-f(x)$ | reflection across x-axis |
| $f(-x)$ | Reflection across y-axis |

List the transformations from its parent function, then **sketch** by hand the graph of each of the given functions. Finally, check your answer using your calculator:

a. $y=(x+3)^2-1$

Transformations:

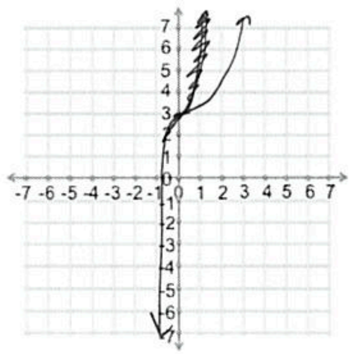
vertical translation down 1
horizontal translation left 3



b. $y=(x-1)^3+4$

Transformations:

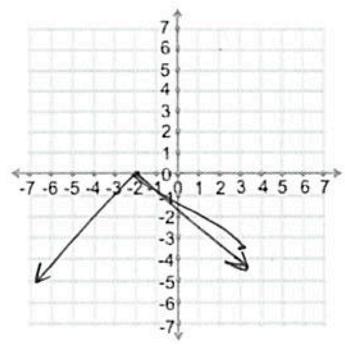
vertical translation up 4
horizontal translation right 1



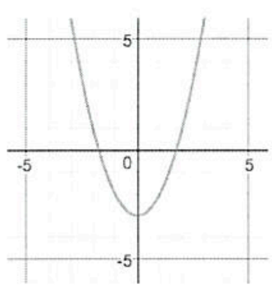
c. $y=-|x+2|$

Transformations:

reflection across x-axis
horizontal translation left 2



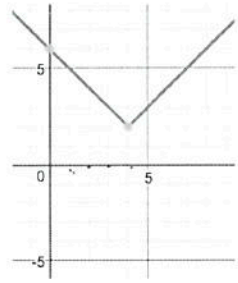
Write an equation for the following graphs:



Parent Function:
$y=x^2$

Transformations:
$y=x^2-3$

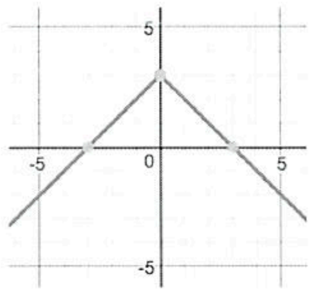
Equation:
vertical translation down three



Parent Function:
$y=|x|$

Transformations:
- horizontal translation right 4
- vertical trans. up 2

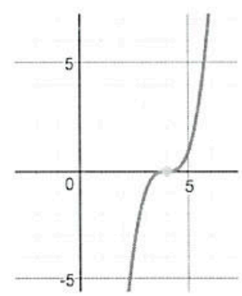
Equation:
$y=|x-4|+2$



Parent Function:
$y=|x|$

Transformations:
reflection over x
up 3

Equation:
$y=-|x|+3$



Parent Function:
$y=x^3$

Transformations:
right 4

Equation:
$y=x^3+4$

**Graph** each of the following on your calculator, and compare with its parent function shown above:

| Function | Transformation |
|---|---|
| $y = 2\lvert x\rvert$ | Vertical stretch by a factor of 2 |
| $y = 3x^3$ | vertical stretch by a factor of 3 |
| $y = \frac{1}{4}x^2$ | vertical compression by a factor of 1/4 |
| $y = \frac{1}{6}\sqrt{x}$ | vertical compression by a factor of 1/6 |
| $y = \lvert 3x\rvert$ | Horizontal compression by a factor of $\frac{1}{3}$ |
| $y = (4x)^2$ | horizontal compression by a factor of 1/4 |
| $y = \lvert \frac{1}{3}x\rvert$ | horizontal stretch by a factor of 3 |
| $y = (\frac{1}{5}x)^3$ | horizontal stretch by a factor of 5 |

**Generalize** the behavior you observed for any function :

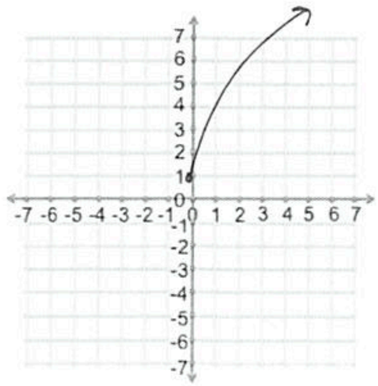
| Graph | Transformation |
|---|---|
| $af(x)$ | Vertical stretch or compression by a factor of a. |
| $f(\frac{1}{b}x)$ | Horizontal stretch or compression by a factor of b |

Given the parent function, apply the following transformations, and give the new function in the space provided:

| Parent Function | Transformations | New Function |
|---|---|---|
| $f(x) = x^2$ | Horizontal stretch by a factor of 3 | $f(x) = (\frac{1}{3}x)^2$ |
| $f(x) = x^3$ | Vertical compression by a factor of 2 | $f(x) = \frac{1}{2}x^3$ |

change to 1/2??

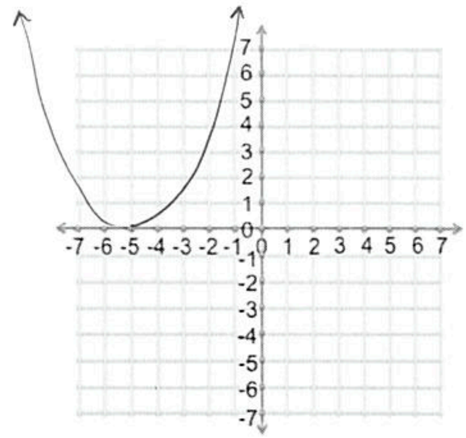
**Sketch** by hand the following transformations. It may be helpful to use your 'table' function in your calculator to do this. Then write a function that describes your transformation.



a) Vertically stretch the square root function by a factor of four, then shift vertically up one unit.

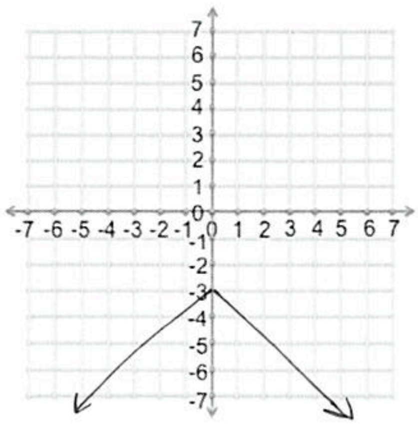
**New Function:** ~~y=...~~ $y = 4\sqrt{x} + 1$

b) Vertically compress the graph by a factor of $\frac{1}{4}$, then shift the cubic function to the left 5 units.



**New Function:** $y = \frac{1}{4}(x+5)^2$

c) Shift the absolute value function down 3 units, then reflect it across the x-axis.



**New Function:** $y = -|x| - 3$

For $f(x) =- 2x^4 + 7x^2 - 4$, **write** the rule for each function:

a) $g(x) = f(x-5)$

b) $g(x) = f(0.25x)$

c) $g(x) = 0.25f(x)$

$$g(x) = -2(0.25x)^4 + 7(0.25x)^2 - 4$$

$$g(x) = -0.0078125x^4 + 0.4375x^2 - 4$$

$$\boxed{g(x) = -2(x-5)^2 + 7(x-5)^2 - 4}$$

$$\boxed{g(x) = -\frac{1}{128}x^4 + \frac{7}{16}x^2 - 4}$$

$$g(x) = 0.25(-2x^4 + 7x^2 - 4)$$

$$\boxed{g(x) = -0.5x^4 + 1.75x^2 - 1}$$